# Introduction to Prob. Theory

## B.Sc I, Paber II (B-195)

## Descriptive Answer Questions. (A)

1. State the classical and statistical def. of Probability. What are basic drawbacks of these definitions?

2. State and Prove addition theorem of Prob. for two events.
   (i) When they are mutually exclusive.
   (ii) When they are not mutually exclusive.

3. State and prove Baye's theorem.

4. In a balt factory machine A, B, C manufacture respectively 30%, 25% and 35% of total of these output 4%, 3% and 2% are defective bolts. A balt is drawn at random from the production and is found defective. What are probabilities that it was manufactured by machine A?

5. The joint prob. function of X and Y is given below. Find marginal and conditional prob. functions of X and Y.

| Y/X | 2 | 4 |
|---|---|---|
| 8 | 0.16 | 0.04 |
| 12 | 0.32 | 0.08 |
| 16 | 0.24 | 0.16 |

6. The joint density function of X and Y is given as:

$$f(x,y) = 2, \quad x<y<1$$
$$= 0 \quad \text{elsewhere}$$

Find the marginal and conditional density functions.

7. A random variable X has the following prob function

| $x$: | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| $f(x)$: | a | 3a | 5a | 7a | 9a | 11a | 13a | 15a | 17a |

(i) ~~Determine~~ (i) Determine the value of a (ii) $P(3<X<5)$ and (iii) CDF.

8 (a) Find whether the following function is a pdf.

$$f(x) = \frac{x^2}{3}, \quad -1 < x < 2$$
$$= 0 \quad \text{elsewhere}$$

Also obtain $P[0 \le x \le 1]$

(b) Define Dist function of a r.v. X. If the dist function of X is given below.

$$F(x) = 1 - e^{-2x}, \quad x > 0$$

find pdf.

9. (a) State and prove addition theorem of mathematical expectation.

(b) Find the variance of linear combination of n random variables.

10. (a) If n dice are tossed and X denotes sum of the numbers on them. Find E(X)

(b) Prove the following

(i) $cov(X+a, Y+b) = cov(X,Y)$, (ii) $cov(aX, bY) = ab\, cov(X,Y)$

11. (a) Define moment generating function of random variable. How are the moments obtained from it?

(b) Obtain MGF for the following pdf and also obtain mean and variance

$$f(x) = \begin{cases} x & \text{for } 0 \le x < 1 \\ 2-x & 1 \le x < 2 \\ 0 & \text{otherwise} \end{cases}$$

12 (a) Define cumulant generating function and obtain interrelationship between cumulants and moments

(b) Except the first cumulant, all cumulants are ind. of change of origin. Also, the cumulants are not independent of change of scale.

13. (a) State and prove central Limit Theorem for iid random variables.

(b) State and prove Chebyshev's inequality.

## Short Answer Type Question (B)

1. Define axiomatic def. of Prob.
2. Define the following
   (a) Mutually exclusive events (b) Mutually exhaustive
   (c) Equally Likely (d) Favourable events
3. Define the following
   (a) Random exp. (b) Ind. events (c) Random variable
   (d) conditional Expectation.
4. Show that $\mathrm{Cov}(X,Y) = E(XY) - E(X)E(Y)$
5. Write down the necessary and sufficient conditions for characteristic function.
6. Obtain the MGF of sum of two i.i.d. random variables
7. Obtain the value of a and E(X) when $f(x) = ax$ $0<x<1$
8. Write the sample space when a coin and a die are thrown simultaneously.
9. Let A and B be two events with $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{4}$ and $P(A \cap B) = \frac{1}{2}$. Find
   (i) $P(A|B)$ (ii) $P(A|\bar{B})$ (iii) $P(\bar{A} \cup \bar{B})$ and (iv) $P(\bar{A}|\bar{B})$
10. Find variance of X and Y, if
    $f(x,y) = x+y, \quad 0 < (x,y) < 1$
11. ~~Explain~~ State the law of large numbers.
12. Obtain the prob distribution whose characteristic function is $\phi(t) = \frac{1}{1+t^2}$
13. Show that $M_{ax+b}(t) = e^{bt} M_x(at)$
14. Write two limitations of M.G.F.
15. Write two advantages of characteristic function over moment generating function.