B.Sc I Code - B-127 Calculus (Limit and continuity)

Q.1. If $f(x) = \frac{x^2 - a^2}{x - a}$, $x \neq a$. Then using (ϵ, δ) definition show that $\lim_{x \to a} f(x) = 2a$.

Q.2. Show that $\lim_{x \to 2} \frac{|x-2|}{x-2}$ does not exists

Q.3. Evaluate the $\lim_{x \to 0} \frac{(1+x)^{1/3} - (1-x)^{1/3}}{x}$

Q.4. Show that the $\lim_{x \to 0} \frac{\sin x}{x} = 1$

Q.5. Let $f(x) = \begin{cases} x & \text{if } x \text{ is rational} \\ -x & \text{if } x \text{ is irrational} \end{cases}$

Show that $\lim_{x \to a}$ exists only when $a = 0$

Q.6 Define left hand limit and Right hand limit of a function at any point a. does the $\lim_{x \to 1} f(x)$ exists for the function defined by

$$f(x) = \begin{cases} x & 0 < x \leq 1 \\ 3 - x & 1 < x \leq 2 \end{cases}$$

Q.7 Define continuity from left & continuity from right

Q.8 Show that the function $f(x)$ defined as

$$f(x) = \begin{cases} 0 & \text{for } x = 0 \\ 1/2 - x & \text{for } 0 < x < 1/2 \\ 1/2 & \text{for } x = 1/2 \\ 3/2 - x & \text{for } \frac{1}{2} < x < 1 \\ 1 & \text{for } x = 1 \end{cases}$$

has three points of discontinuity find them and also draw the graph of the function.

Q.9 Examine the function for continuity at $x = 0$; where $f(x) = \frac{\sin^2 ax}{x^2}$ for $x \neq 0$, $f(0) = 1$ for $x = 0$.

Q.10 Prove that the function $f(x) = \frac{|x|}{x}$ for $x \neq 0$ $f(0) = 0$ is continuous at all points except $x = 0$.

Q.11 Show that the function defined by $f(x) = \frac{x e^{1/x}}{1 + e^{1/x}}$ for $x \neq 0$ $f(0) = 1$ is not continuous at $x = 0$ and also show how this discontinuity can be removed.

Q.12 Examine the function for continuity at $x = 0$ and $x = 1$

$$f(x) = \begin{cases} x^2 & \text{for } x \leq 0 \\ 1 & \text{for } 0 < x \leq 1 \\ 1/x^2 & \text{for } x > 1. \end{cases}$$

Q.13 Is the function $f(x) = x \cos(1/x)$ if $x \neq 0$ and $f(0) = 0$ continuous.

Q.14. If $f(x)$ and $g(x)$ are continuous at $x = a$ then show that the function $(f \cdot g)(x)$ is also continuous.

Q.15. If the $\lim_{x \to a} f(x) = l$ and $\lim_{x \to a} g(x) = m$ then find the ~~lim~~ $\lim (fg)(x)$?

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