B.Sc. I. Code B.128 Geometry and Vector Calculus

(Vectors)

Q.1 If $\bar{r} = (t+1)i + (t^2+t+1)j + (t^3+t^2+t+1)k$, find $\frac{dr}{dt}$ and $\frac{d^2 r}{dt^2}$.

Q.2. If $\bar{r} = \sin t\, i + \cos t\, j + t k$, find the values of
(i) $\frac{d\bar{r}}{dt}$ (ii) $\frac{d^2 \bar{r}}{dt^2}$ (iii) $\left|\frac{d\bar{r}}{dt}\right|$ (iv) $\left|\frac{d^2\bar{r}}{dt^2}\right|$

Q.3 If $\bar{a}, \bar{b}$ are constant vectors, w is a constant and $\bar{r}$ is a vector function of the scalar variable t given by
$\bar{r} = \cos wt\, a + \sin wt\, b$, show that
(i) $\frac{d^2\bar{r}}{dt^2} + w^2 \bar{r} = \bar{0}$ (ii) $\bar{r} \times \frac{d\bar{r}}{dt} = w\, \bar{a} \times \bar{b}$.

Q.4. Find the value of $\bar{r}$ satisfying the equation $\frac{d^2\bar{r}}{dt^2} = \bar{a}$, where $\bar{a}$ is a constant vector. Also it is given that when $t=0$, $\bar{r} = \bar{0}$ and $\frac{d\bar{r}}{dt} = \bar{u}$.

Q.5 Given that $\bar{r}(t) = \begin{cases} 2i - j + 2k, & \text{when } t = 2 \\ 4i - 2j + 3k & \text{when } t = 3, \end{cases}$
show that $\int_2^3 \left(\bar{r} \times \frac{d\bar{r}}{dt}\right) dt = 10$

Q.6 If vector $\bar{a}(t)$ has constant direction then:
$\bar{a} \times \frac{d\bar{a}}{dt} = \bar{0}$.

Q.7 If $\bar{r} = e^{nt}\bar{a} + e^{-nt}\bar{b}$, where $\bar{a}$ and $\bar{b}$ are constant vectors, show that. $\frac{d^2\bar{r}}{dt^2} - n^2\bar{r} = \bar{0}$

Q.8. Prove that if $\bar{a}$ has constant length, then $\bar{a}$ and $\frac{d\bar{a}}{dt}$ are perpendicular provided $\left|\frac{d\bar{a}}{dt}\right| \neq 0$.

Q.9. If $\bar{a}$ is a differentiable vector function of scalar variable t, then
$$\frac{d}{dt}\left(\bar{a} \times \frac{d\bar{a}}{dt}\right) = \bar{a} \times \frac{d^2\bar{a}}{dt^2}.$$

Q.10 If $\bar{r}(t) = 5t^2 i + tj - t^3 k$ prove that
$$\int_1^2 \left(\bar{r} \times \frac{d^2\bar{r}}{dt^2}\right) dt = -14i + 75j - 15k.$$

Q.11 If $f(x,y,z) = 3x^2y - y^3z^2$ find grad f at the point $(1,-2,-1)$

Q.12 if $r = |\bar{r}|$, where $\bar{r} = xi + yj + zk$, prove that
(i) $\nabla r = \frac{1}{r}\bar{r}$ (ii) $\nabla\left(\frac{1}{r}\right) = -\frac{\bar{r}}{r^3}$

Q.13 Prove that
(i) div $\bar{r} = 3$ (ii) Curl $\bar{r} = \bar{0}$

Q.14 Prove that
(i) div $(\bar{A} \times \bar{B}) = \bar{B} \cdot$ curl $\bar{A} - \bar{A} \cdot$ Curl $\bar{B}$

Q.15 If $\nabla^2 f(r) = 0$, show that $f(r) = \frac{c_1}{r} + c_2$, where $r^2 = x^2+y^2+z^2$ and c_1 and c_2 are arbitrary constants.

Q. 16 Prove that
(i) $\nabla^2\left(\frac{1}{r}\right) = 0$ (ii) div grad $r^n = n(n+1) r^{n-2}$.

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