B.Sc. III B. 326 Analysis

Topic. Convergence of improper Integral

Q.1. The definite integral $\int_a^b f(x)\,dx$ is said to be a proper integral if
(a) $f(x)$ is bounded (b) range of integration is finite
(c) either (a) or (b) (d) both (a) and (b)

Q.2. The definite integral $\int_a^b f(x)\,dx$ is said to be improper integral if
(a) $f(x)$ is unbounded (b) interval (a,b) is not finite
(c) $f(x)$ is unbounded and interval (a,b) is not finite
(d) any one of the above.

Q.3. A real valued function f defined on an interval I is said to monotonically increasing if
(a) $x > y \Rightarrow f(x) > f(y)\ \forall x, y \in I$ (b) $x > y \Rightarrow f(x) < f(y)\ \forall x, y \in I$
(c) $x < y \Rightarrow f(x) > f(y)\ \forall x, y \in I$ (d) $x > y \Rightarrow f(x) \le f(y)\ \forall x, y \in I$

Q.4. If in the definite integral $\int_a^b f(x)\,dx$, the range of integral is finite and function $f(x)$ is bounded then $\int_a^b f(x)\,dx$ is:
(a) Improper integral of first kind (b) Proper integral
(c) Improper integral of second kind (d) none of these

Q.5. The integral $\int_0^\infty \frac{dx}{1+x^2}$ is an improper integral of the:
(a) first kind (b) second kind
(c) Neither (a) nor (b) (d) none of these

Q.6. For the integral $\int_0^\infty e^{-mx}\,dx$ $(m>0)$, which is/are correct:
(a) The value of integral is $1/m$ (b) The integral is convergent
(c) both (a) & (b) (d) none of these

Q.7. The integral $\int_0^\infty \frac{x^{a-1}}{1+x}\,dx$ is:
(a) convergent if $0<a<1$ (b) divergent if $a\geq 1$
(c) divergent if $a \le 0$ (d) All of the above

Q.8. The integral $\int_0^4 \frac{dx}{(x-2)(x-3)}$ is an improper integral of the
(a) first kind (b) second kind
(c) Neither (a) nor (b) (d) none of these

Q.9. The integral $\int_0^1 x^{m-1}(1-x)^{n-1}\,dx$ is convergent when.
(a) $m>0$ (b) $n>0$ (c) $m>0, n>0$ (d) none of these

Q.10 By comparison test integral $\int_0^1 \frac{\sec x}{x}\,dx$ is divergent because
(a) $\int_0^1 \sec x\,dx$ is convergent (b) $\int_0^1 \frac{1}{x}\,dx$ is convergent
(c) $\int_0^1 \sec x\,dx$ is divergent (d) $\int_0^1 \frac{1}{x}\,dx$ is divergent

Q.11 If $\int^{\infty} |f(x)|\,dx$ is convergent then $f(x)$ is said to be
(a) uniformly convergent (b) divergent
(c) absolutely convergent (d) oscillatory

Q.12 If the $\lim_{x\to\infty} \frac{f(x)}{g(x)}$ is a definite number, other than zero, the $\int_a^{\infty} f(x)\,dx$ and $\int_a^{\infty} g(x)\,dx$ both are:
(a) convergent (b) divergent
(c) either both convergent or divergent (d) none of these

Q.13 The integral $\int_0^{\infty} e^{-x} x^{n-1}\,dx$ is convergent if:
(a) $n=0$ (b) $n>0$ (c) $n<0$ (d) none of these

Q.14 The value of μ in the μ-test of convergency for the integral $\int_0^{\infty} \frac{x\,dx}{(1+x)^3}$ is:
(a) -2 (b) 2 (c) 1 (d) $=3$

Q.15 If $\int_a^{\infty} f(x)\,dx$ is convergent and $\phi(x)$ is bounded and monotonic for $x>a$ then $\int_a^{\infty} f(x)\phi(x)\,dx$ is:
(a) divergent (b) convergent
(c) bounded (d) monotonic.

Q.16 For the integral $\int_{-\infty}^{\infty} \frac{dx}{x^2+2x+2}$ which is/are true:
(a) The value of the integral is π (b) the integral is convergent
(c) both (a) & (b) are true (d) none of these

Q.17 For the integral $\int_0^1 \frac{dx}{\sqrt{1-x}}$ which is/are true:
(a) $x=1$ is the point of infinite discontinuity
(b) given integral is convergent
(c) The value of the integral is 2
(d) All are true

Q.18 The integral $\int_3^{\infty} \frac{dx}{(x-2)^2}$ is
(a) convergent (b) divergent
(c) may or may not be convergent (d) none of these

Q.19 Let $f(x)$ be a bounded and integrable in the interval $[a,\infty]$, where $a>0$, such that $\lim_{x\to\infty} x^{\mu} f(x)$ exists the $\int_a^{\infty} f(x)\,dx$ is convergent if:
(a) $\mu>1$ (b) $\mu\geq 1$ (c) $\mu<1$ (d) $\mu\leq 1$