B.Sc. I B-126 Algebra & Trigonometry

Topic - Group

Q.1. Define Binary operation on a set with ~~one~~ example.

Q2. Define Semi-group and group also define Abelian group.

Q3. Let R be the set of all real numbers and * be a binary operation on R defined by $a * b = a + b - ab$. Determine identity element in R and also find the inverse of $a \in R$.

Q4. Show that the set Q_+ of all positive rational numbers forms an abelian group under the binary operation defined by $a * b = \frac{ab}{2}$

Q5. Show that the inverse of the product of two elements of a group G is the product of the inverses taken in the reverse order. i.e. $(ab)^{-1} = b^{-1} a^{-1}$ $\forall a, b \in G$.

Q6 Show that the four fourth roots of unity $1, -1, i, -i$ form an abelian group w.r.t multiplication. Use composition table for it.

Q7. Prove that $G = \{1, 5, 7, 11\}$ is an abelian group under multiplication modulo 12.

Q.8. Define odd & Even permutations. Examine whether the following permutation is even or odd.

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 8 & 5 & 3 & 6 & 2 & 1 & 4 & 9 & 7 \end{pmatrix}$$

Q.9. Define order of a group and order of an element of a group.

Q.10. Find the order of each element of the group $G = \{1, -1, i, -i\}$ with respect to multiplication.

Q11 Prove that the order of ~~each~~ an element a of a group is the same as that of its inverse a^{-1}.

Q12. ~~Define~~ Define complex and subgroup of a group.

Q.13. A necessary and sufficient condition for a complex H of a group G to be a subgroup is that
$a \in H, b \in H \Rightarrow ab^{-1} \in H$ where b^{-1} is the inverse of b in G.

Q.14 If H and K are two subgroups of G, then HK is a subgroup of G iff $HK = KH$

Q.15 If H and K are two subgroups of G, then $H \cap K$ is also a subgroup of G.

Q.16. If H and K are two subgroup of G, then $H \cup K$ is subgroup of G if either $H \subseteq K$ or $K \subseteq H$.

Q.17 Define coset of a ~~group~~ subgroup in a group.

Q.18 Show that any two right cosets of a subgroup are either disjoint or identical.

Q.19. State and prove Lagrange's theorem for group.

Q20. Define Cyclic group and define generator of a ~~gro~~ cyclic group.

Q21. Prove that every cyclic group is abelian group.

Q22. Show that the group $\{(1, 2, 3, 4), \times_5\}$ is cyclic.