B.Sc.I. B-127 Calculus Topic: Differentiability

Q.1. Show that continuity is a necessary but not a sufficient condition for the existence of a finite derivative.

Q.2. Give an example to show that function may be continuous at a point but not differentiable at that point.

Q.3. Prove that the function $f(x)=|x|$ is continuous at $x=0$ but differentiable at $x=0$, also draw the graph of the function.

Q.4 Draw the graph of the function $y=|x-1|+|x-2|$ in the interval $[0,3]$ and discuss the continuity and differentiability of the function in this interval.

Q.5. State Rolle's theorem and verify it for the function $f(x)=e^x \sin x$ for $a=0$ and $b=\pi$.

Q.6. State Lagrange's mean value theorem.

If $f(x)=(x-1)(x-2)(x-3)$ and $a=0, b=4$ find c using Lagrange's mean value theorem.

Q.7 State Cauchy's mean value theorem. If $f(x)=e^x$ and $g(x)=e^{-x}$ then find c of Cauchy mean value theorem.

Successive differentiation

Q.8. State Leibnitz's theorem and find nth differential coefficient of $x^3 \log x$.

Q.9. If $y=x^2 e^x$ show that $y_n=\frac{1}{2}n(n-1)y_2-n(n-2)y_1+\frac{1}{2}(n-1)(n-2)$

Q.10. If $y=x^n \log x$, prove that $x y_{n+1}=n!$

Q.11. If $y=(\sin^{-1}x)^2$, prove that $(1-x^2)y_2-xy_1-2=0$, and also prove $(1-x^2)y_{n+2}-x(2n+1)y_{n+1}-n^2 y_n=0$.

Q.12 If $y=(x^2-1)^n$, prove that $(x^2-1)y_{n+2}+2xy_{n+1}-n(n+1)y=0$

Q.13 If $y=\sin^{-1}x$, prove that

$$(1-x^2)y_{n+2}-(2n+1)x y_{n+1}-n^2 y_n=0$$

and hence find the value of $(y_n)_0$.

Q.14. If $y=\left[\log\{x+\sqrt{(1+x^2)}\}\right]^2$, prove that $(y_{n+1})_0=-n^2(y_n)_0$, hence find $(y_n)_0$?

Q.15 If $y=e^{a\cos^{-1}x}$ prove that

$$(1-x^2)y_{n+2}-(2n+1)x y_{n+1}-(n^2+a^2)y_n=0.$$

and hence find the value of y_n for $x=0$.