B.Sc I · B-127 Calculus Topic - integral Calculus

Q.1 Prove that $B(m,n) = \frac{\Gamma m\,\Gamma n}{\Gamma(m+n)}$, where $m>0$, $n>0$

Q.2 Prove that $\int_0^{\pi/2} \cos^{m}\theta \sin^{n}\theta\, d\theta = \frac{\Gamma\frac{m+1}{2}\,\Gamma\frac{n+1}{2}}{2\,\Gamma\frac{m+n+1}{2}}$, $m>-1$, $n>-1$.

Q.3 Prove that $\Gamma m\,\Gamma(m+\tfrac{1}{2}) = \frac{\sqrt{\pi}}{2^{2m-1}}\Gamma 2m$, where $m>0$.

Q.4 Show that $\int_0^{\infty} \cos(b z^{1/n})\, dz = \frac{1}{b^n}\Gamma(n+1) \cos\frac{n\pi}{2}$.

Q.5 Show that $\int_0^{2} x^4 (8-x^3)^{-1/3} dx = \frac{16}{3} B\left(\frac{5}{3},\frac{2}{3}\right)$.

Q.6 $\int_0^1 (1-x^4)^{1/4} dx = \frac{1}{4}\frac{(\Gamma\frac{1}{4})^2}{2\Gamma\frac{2}{4}}$.

Q.7 Evaluate $\int_0^{\pi}\int_a^{a(1+\cos\theta)} r^2 \cos\theta \, d\theta\, dr$.

Q.8 Evaluate $\iiint z^2\, dx\,dy\,dz$ over the sphere $x^2+y^2+z^2=1$.

Q.9 Change the order of integration in

$$\int_0^a \int_{\sqrt{a^2-x^2}}^{x+2a} f(x,y)\, dx\, dy.$$

Q.10 Find the area included between the curves $y^2=4ax$ and $x^2=4ay$.

Q.11 Show that the larger of the two areas into which the circle $x^2+y^2=64a^2$ is divided by the parabola $y^2=12ax$ is $\frac{16}{3}a^2[8\pi-\sqrt{3}]$.

Find the area common to the two curves $y^2=ax$, $x^2+y^2=4ax$.

Q.12 Find the area of the cardioid $r=a(1+\cos\theta)$.

Q.13 Find the area of the arc of the parabola $y^2=4ax$ extending

Q.15 Find the length of the arc to an extremity of the latus rectum.

from the vertex to

Q.16 Show that the intrinsic equation of the cycloid $x=a(t+\sin t)$, $y=a(1-\cos t)$ is $s=4a\sin\psi$. Hence find the length of the complete cycloid.

Q.17 Find the intrinsic equation of the equiangular spiral ~~$p = r\sin\alpha$~~. $p = r\sin\alpha$.

Q.18 The cardiode $r=a(1+\cos\theta)$ revolves about the initial line, find the volume of solid thus generated.

Q.19 Find the curve surface of a hemisphere of radius a.

Q.20 Find the surface generated by the revolution of an arc of the catenary $y=c\cosh(x/c)$ about the axis of x.

Q.21 Find the surface of the solid generated by the revolution of the ellipse $x^2+4y^2=16$ about its major axis.

Q.22 Show that the volume generated by the revolution of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ about the line $x=2a$ is $4\pi^2a^2b$.