B.Sc. I Code-B-126 Algebra & Trigonometry

Topic- Normal Subgroup & Ring

Q.1. Define Simple group and Normal Sub group. Show that A Sub group H of a group G is normal if and only if.

$$xHx^{-1} = H \quad \forall x \in G$$

Q2. Show that a subgroup H of a group G is normal subgroup of G iff the product of two right cosets of H in G is again a right coset of H in G.

Q.3. Define conjugate elements in a group. the relation of conjugacy is an equivalence relation on G.

Q.4. Define Normalizer of an element of a group and show that the Normalizer N(a) of $a \in G$ is a subgroup of G.

Q5. Define Centre of a group and Prove that the centre Z of a Group G is a normal sub group.

Q6. If f is a homomorphism of a group G into a group G', then

(i) $f(e) = e'$, where e is the identity of G and e' is the identity of G'.

(ii) $f(a^{-1}) = [f(a)]^{-1}$ $\forall a \in G$.

Q.7 Define Kernel of a Homomorphism and If f be a homomorphism of group G into a group G' with kernel K, then show that K is a normal subgroup of G.

Q.8 Define Integral Domain & Field. Give an example of an Integral domain which is not a field.

Q.9 Prove that every field is an Integral Domain.

Q.10. Show that the left Ideal generated by $I_1 \cup I_2$ of two left Ideals is the set $I_1 + I_2$ consisting of the elements of R obtained by adding any element of I_1 to any element of I_2.

Q.11 Define proper and improper Ideals and show that a field has no proper Ideals.

Q12. A Ring R is without zero divisors, if and only if both cancellation laws hold in R.

Q.13 Prove that the subset S of all matrices of the form $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ with a and b integers, form a subring of the ring of all 2×2 matrices having elements as integers. Also show that S is neither a right ideal nor a left ideal of R.

Q14. Show that $(F, +, \cdot)$ is a field if

$$F = \{a + b\sqrt{2};\ a, b \in Q\}$$

Q.15 If S_1 and S_2 be two ideals of a Ring R and $S_1 + S_2 = \{s_1 + s_2 : s_1 \in S_1 \text{ and } s_2 \in S_2\}$ then show that $S_1 + S_2$ is an Ideal of R.