B.Sc. I - B.127 Calculus Topic. Differential Calculus

Q.1. Expand $\log(1+x)$ by Maclaurin's theorem.

Q.2. Expand $\sin x$ in powers of $(x-\pi/2)$ by using Taylor's series.

Q.3. Evaluate $\lim_{x\to 0} \dfrac{x\cos x - \log(1+x)}{x^2}$

Q.4. Evaluate $\lim_{x\to\infty} \left[\dfrac{\log x}{x}\right]^{1/x}$

Q.5. If $u = \log(x^3+y^3+z^3-3xyz)$. Show that

$$\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 u = -\frac{9}{(x+y+z)^2}$$

Q.6 If $u = xy\,f(y/x)$, then find the value of $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y}$?

Q.7. State Euler's theorem. ~~and verify it for the function~~

If $u = \sin^{-1}\left\{\dfrac{x^2+y^2}{x+y}\right\}$, Show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \tan u$.

Q.8 If $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$, Show that

$$\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)} = r^2\sin\theta.$$

Q.9. Show that the functions

$u = x^2+y^2+z^2$; $v = x+y+z$ and $w = xy+yz+zx$ are not independent. Also find the relation between them.

Q.10 Examine the function $z = x^2y - y^2x - x + y$ for maximum and minimum value. also find the saddle points of the given surface.

Q.11 Show that the parabolas $r = a/(1+\cos\theta)$ and $r = b/(1-\cos\theta)$ intersect orthogonally.

Q.12 Show that the pedal equation of the parabola $y^2 = 4a(x+a)$ is $p^2 = ar$.

Q.13 For the Cardioid $r = a(1-\cos\theta)$ show that

(i) $\phi = \theta/2$ (ii) $p = 2a\sin^3\theta/2$ (iii) the pedal Equation is $2ap^2 = r^3$.

Q.14. If CP, CD be a pair of conjugate semi-diameters of an ellipse prove that the radius of curvature at P is CD^3/ab, a and b being the length of the semi-axes of the ellipse.

Q.15. Show that for the cardioid $r = a(1+\cos\theta)$, $\rho = \frac{2}{3}\sqrt{2ar}$

Q.16 Find the co-ordinates of the centre of curvature for the point (x, y) on the parabola $y^2 = 4ax$.