7.2.3 Response of a Narrowband Filter to Noise

If the representation of noise as a superposition of spectral components is a reasonable one, we should expect that when noise is passed through a narrowband filter the output of the filter should look rather like a sinusoid. We find that such is indeed the case, for we find that the output of a narrowband filter with noise input has the appearance shown in Fig. 7.4. The output waveform looks like a sinusoid except that, as expected, the amplitude varies randomly. The spectral range of the *envelope* of the filter output encompasses the spectral range from $-B/2$ to $B/2$, where B is the filter bandwidth. The average frequency of the waveform is the center frequency f_c of the filter. If $B \ll f_c$ the envelope changes very "slowly" and makes an appreciable change only over many cycles. Thus, while the spacings of the *zero crossings* of the waveform are not precisely constant, the change from cycle to cycle is small and when averaged over many cycles is quite constant at the value $1/2f_c$. Finally, we may note that as B becomes progressively smaller, so also does the average amplitude, and the waveform becomes more and more sinusoidal.

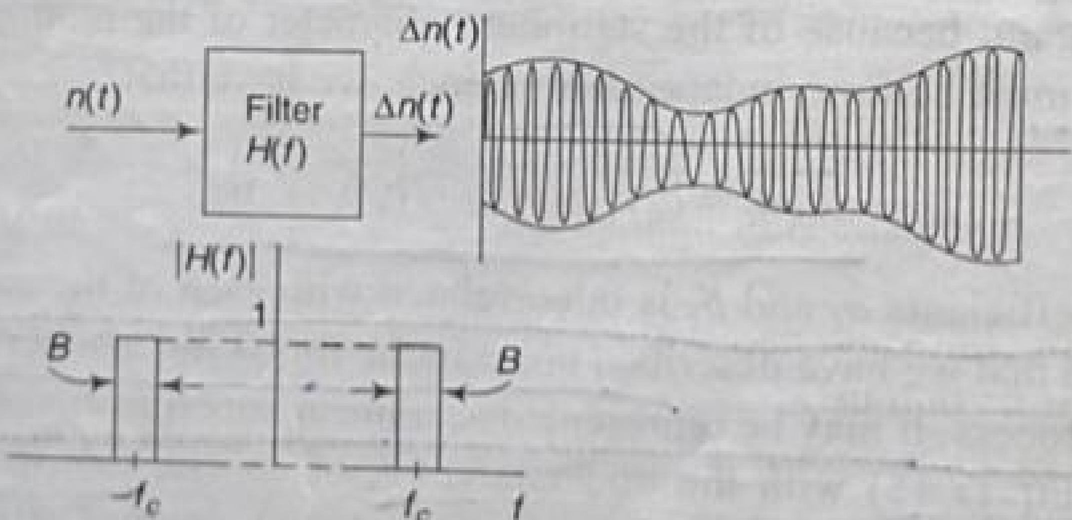

Fig. 7.4 Response of a narrow and filter to noise.

7.2.4 Effect of a Filter on PSD of Noise

Let a spectral component of noise $n_{k_i}(t)$ given by Eq. (7.15) be the input to a filter whose transfer function at the frequency $k\,\Delta f$ is

$$\boxed{H(k\,\Delta f) = |H(k\,\Delta f)|e^{j\varphi_k} = |H(k\,\Delta f)|\angle\varphi_k} \tag{7.29}$$

The corresponding output spectral component of noise will be $n_{k_o}(t)$

$$n_{k_o}(t) = |H(k\,\Delta f)|\,a_k \cos(2\pi k\,\Delta f t + \varphi_k) + |H(k\,\Delta f)|\,b_k \sin(2\pi k\,\Delta f t + \varphi_k) \tag{7.30}$$

The power P_{k_i} associated with $n_{k_i}(t)$ is, from Eq. (7.20),

$$P_{k_i} = \frac{\overline{a_k^2} + \overline{b_k^2}}{2} \tag{7.31}$$

Since $|H(k\,\Delta f)|$ is a deterministic function, $\overline{[|H(k\,\Delta f)|a_k]^2} = |H(k\,\Delta f)|^2\,\overline{a_k^2}$, and $\overline{[|H(k\,\Delta f)|b_k]^2} = |H(k\,\Delta f)|^2\,\overline{b_k^2}$. Hence, comparing Eq. (7.30) with Eq. (7.15), we find that the power P_{k_o} associated with $n_{k_o}(t)$ is

$y(t) = n_k(t)$

$n_k(t) = a_k \cos 2\pi k \Delta f t + b_k \sin 2\pi k \Delta f t$

$$P_{k_o} = |H(k\,\Delta f)|^2\,\frac{\overline{a_k^2} + \overline{b_k^2}}{2} \tag{7.32}$$

Finally, then, from Eqs (7.31) and (7.32), using also Eq. (7.20), we have that the power spectral densities at input and output, $G_{n_i}(k\,\Delta f)$ and $G_{n_o}(k\,\Delta f)$, are related by

$$G_{n_o}(k\,\Delta f) = |H(k\,\Delta f)|^2\,G_{n_i}(k\,\Delta f) \tag{7.33}$$

In the limit as $\Delta f \to 0$ and $k\,\Delta f$ is replaced by a continuous variable f, Eq. (7.33) becomes

$$\boxed{G_{n_o}(f) = |H(f)|^2\,G_{n_i}(f)} \tag{7.34}$$

Note the similarity between this result and Eq. (1.81), which applies to a deterministic waveform.

We got this relation by convolution for a transmission of a random process through a linear system in Sec. 6.5.6.