7.6 REPRESENTATION OF NOISE USING ORTHONORMAL COORDINATES

In our discussion of the frequency-domain representation of noise we saw that a noise process can be represented as a sum of orthonormal functions. These orthonormal functions are the sines and cosines. In our discussion of the Gram–Schmitt technique, where our interest concerned a waveform defined over a time interval T, we pointed out that limitless other functions, orthonormal over T, are also possible. We consider here some features of the representation of noise in terms of orthonormal functions.

If $u_i(t)$ are a set of orthonormal functions in the interval T then in that interval, noise $n(t)$ is

$$n(t) = \sum_{i=0}^{\infty} n_i u_i(t) \tag{7.102}$$

in which n_i is the coefficient of the ith component and is evaluated in the usual manner, that is,

$$n_i = \int_0^T n(t)u_i(t)\,dt \tag{7.103}$$

If the noise $n(t)$ is a Gaussian random process with a mean value of zero then n_i is a Gaussian random variable with a zero mean value.

We shall now determine the correlation between coefficients say n_i and n_j. We have

$$n_j n_i = \int_0^T n(t)u_i(t)\,dt \int_0^T n(\lambda)u_j(\lambda)\,d\lambda \tag{7.104a}$$

$$= \int_0^T dt \int_0^T d\lambda n(t)n(\lambda)u_i(t)u_j(\lambda) \tag{7.104b}$$

where t and λ are dummy variable of integration. We now take the ensemble average of both sides of Eq. (7.104b). Interchanging the order of averaging and integrating as in Sec. 7.4, we have

$$E(n_i n_j) = \int_0^T dt \int_0^T d\lambda E[n(t)n(\lambda)]u_i(t)u_j(\lambda) \tag{7.105}$$

Since the noise process is ergodic, Eq. (6.141) applies and we have that the autocorrelation of the process is

$$R(t-\lambda) = E[n(t)n(\lambda)] \tag{7.106}$$

Further, assuming white noise of power spectral density $G(f) = \eta/2$ we have, as in Eq. (7.73) that

$$R(t-\lambda) = \eta/2\ \delta(t-\lambda) \tag{7.107}$$

From Eqs (7.104), (7.105), and (7.106) we have that

$$E(n_i n_j) = \int_0^T dt \int_0^T d\lambda\ \eta/2\ \delta(t-\lambda)u_i(t)u_j(\lambda) \tag{7.108}$$

$$= \eta/2 \int_0^T u_i(t)u_j(t)\,dt = \begin{cases} \eta/2 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases} \tag{7.109}$$

We shall have occasion in Sec. 11.2 in our study of the probability of error to make use of the fact that the additive white Gaussian noise $n(t)$ can be represented as Eq. 7.102 where the n_i form a set of statistically independent random variables, each with a mean of zero and a variance of $\eta/2$.